

Triple Electroweak Gauge-Boson Production at Fermilab Tevatron Energies

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Abstract

We calculate the three gauge-boson production in the Standard Model at Fermilab Tevatron energies. At $\sqrt{s} = 2$ TeV in $p\bar{p}$ collisions, the cross sections for the triple gauge-boson production are typically of order 10 femtobarns (fb). For the pure leptonic final states from the gauge-boson decays and with some minimal cuts on final state photons, the cross sections for $p\bar{p} \rightarrow W^\pm \gamma \gamma, Z \gamma \gamma$ and $W^+ W^- \gamma$ processes are of order a few fb, resulting in a few dozen clean leptonic events for an integrated luminosity of 10 fb^{-1} . The pure leptonic modes from other gauge-boson channels give significantly smaller rate. Especially, the trilepton modes from $W^+ W^- W^\pm$ and $t\bar{t} W^\pm \rightarrow W^+ W^- W^\pm$ yield a cross section of order 0.1 fb if there is no significant Higgs boson contribution. For a Higgs boson with $m_H \simeq 2M_Z$, the triple massive-gauge-boson production rate could be enhanced by a factor of 4 – 6.

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I. INTRODUCTION

Ever since the experimental discovery of the electroweak vector bosons W^\pm and Z (generically denoted by V henceforth, unless specified otherwise) at the CERN $\bar{p}p$ collider, studies on their production in collider experiments have provided important means to examine the electroweak Standard Model (SM), and to explore new physics beyond the SM [1]. The production of two gauge-bosons [2] at hadron and e^+e^- colliders may provide a crucial test of the non-Abelian three-gauge-boson interactions [3], because of intimate cancellation in the SM between the contributions of gauge-boson exchanges in the s -channel and fermion exchanges in the t - and u -channels. The production of three gauge-bosons [4–6] involves four-gauge-boson interactions and should be sensitive to these quartic vertices [7]. Recent experimental studies of gauge-boson pair events at hadron colliders have set limits on anomalous couplings for the three-gauge-boson interactions from the direct production channels [8]. Further improvements on those studies are anticipated from future experiments with the Tevatron Main Injector (an annual integrated luminosity of 1 fb^{-1}). A possible further upgrade of the Tevatron (a luminosity of order $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ [9] or about $10 \text{ fb}^{-1}/\text{yr}$, and a possible energy upgrade to 4 TeV [10]), especially the LHC (a luminosity of $10 - 100 \text{ fb}^{-1}$ [11]) may also add to these studies [12]. Experiments at LEP II will study the process $e^+e^- \rightarrow W^+W^-$ in great detail [13]. The future e^+e^- Linear Colliders will explore not only the processes of pair gauge-boson production, but also the triple gauge-boson production [14,7]. Furthermore, since heavy particles often decay to gauge-bosons as a final state, it is important to examine the multi-gauge-boson production in searching for new physics. The most notable example is that the recent studies on $WW(\rightarrow l\nu, l\nu)+2\text{-jet}$ and $W(\rightarrow l\nu)+4\text{-jet}$ events at the Fermilab Tevatron have resulted in the discovery of the elusive top quark [15]. Other examples include a heavy Higgs boson decay $H^0 \rightarrow W^+W^-, ZZ$ [11,16]; heavy chargino and neutralino decays and squark/gluino cascade decays in supersymmetric theories [17]; longitudinal gauge-boson strong scattering [18]; new heavy gauge-boson decays [19]; and other exotic heavy particles that may decay to electroweak gauge-bosons [20].

Motivated by the great success of the Fermilab Tevatron experiments, we carry out the calculation for three electroweak-gauge-boson production in the SM for the energy range $1.8 - 4$ TeV, relevant to future experiments at a Tevatron upgrade for luminosity and center of mass energy [9,10]. We have considered all combinations for the three electroweak-gauge-boson processes (except for the pure QED process of three-photon production):

$$p\bar{p} \rightarrow W^\pm \gamma \gamma, Z \gamma \gamma, \quad (1)$$

$$p\bar{p} \rightarrow W^+ W^- \gamma, W^\pm Z \gamma, ZZ \gamma, \quad (2)$$

$$p\bar{p} \rightarrow W^+ W^- W^\pm, W^+ W^- Z, W^\pm ZZ, ZZZ \quad (3)$$

Since a top quark almost exclusively decays to a $W^+ b$ final state, there will be significant contribution to $W^+ W^-$ production from $t\bar{t}$ decays. We therefore also include the top-quark channels

$$p\bar{p} \rightarrow t\bar{t} \gamma, t\bar{t} W^\pm, t\bar{t} Z \quad \text{with} \quad t\bar{t} \rightarrow W^+ W^-. \quad (4)$$

These processes of Eqs. (1–4) have rather small production cross sections at Tevatron energies, typically being of order 10 fb, about 100 times smaller than the gauge-boson pair production. However, as we will see, it is possible to observe those processes at an upgraded Tevatron. They must be quantified in order to explore the quartic gauge-boson self-interactions. Moreover, they need to be reliably estimated in searching for new physics, such as in the multi-charged lepton channels in SUSY searches [17].

In Sec. II, we present the cross sections of the processes in Eqs. (1–4). In Sec. III, we discuss the results with leptonic final states from the gauge-boson decays and some typical kinematical distributions. We finally summarize our results in Sec. IV.

II. CROSS SECTIONS FOR TRIPLE GAUGE-BOSON PRODUCTION AT TEVATRON ENERGIES

The processes listed in Eqs. (1–4) have been individually studied in the literature, but only in pp collisions at Supercollider energies. The processes $W^\pm \gamma \gamma$ [21] and $Z \gamma \gamma$ [22] in

Eq. (1) have been calculated as backgrounds to the intermediate Higgs boson signal for $H^0 \rightarrow \gamma\gamma$. These in Eq. (2) were first evaluated in Ref. [5] and those of Eq. (3) in Refs. [4,5]. Calculations for the $t\bar{t}W^\pm$, $t\bar{t}Z$ and $t\bar{t}\gamma$ processes first appeared in Refs. [23], [24] and [25], respectively. We have been able to reproduce the total cross sections in the literature whenever available to compare.

In this section, we present the total cross sections for all the production processes in Eqs. (1–4) in $p\bar{p}$ collisions at Tevatron energies. In our numerical calculations, we have adopted the helicity amplitude techniques, following the schemes in Refs. [26] and [27]. We have chosen the following input mass parameters [28]: $M_W = 80.22$ GeV, $M_Z = 91.187$ GeV, $m_t = 175$ GeV, and $\alpha_{em} = 1/128$. We use the parton distribution functions Set A of Martin-Roberts-Stirling [29]. For the processes of Eqs. (1–3), the factorization scale is set at the parton c. m. energy $\sqrt{\hat{s}}$; while for the processes of Eq. (4), we set the renormalization scale in α_s and the factorization scale both at twice of the top-quark mass $2m_t$.

Figure 1 shows the total cross sections for processes with one or two direct photons, Eqs. (1–2). To avoid the infrared divergence for the final state photons in our tree-level calculation and to roughly simulate the experimental detector coverage, we have imposed some minimal cutoff on the transverse momentum (p_T^γ) and the pseudorapidity (η^γ) as

$$p_T^\gamma > 10 \text{ GeV}; \quad \text{and} \quad |\eta^\gamma| < 2.5, \quad (5)$$

where $\eta^\gamma = \ln \cot(\theta^\gamma/2)$, with θ^γ the polar angle of the photon in the parton c. m. frame with respect to the proton direction. No cuts are applied to the W^\pm and Z throughout this paper; but we will comment in Sec. II on the effects if experimental acceptance cuts are imposed. We see that at $\sqrt{s}=2$ TeV, the total cross sections are about 7 fb for $ZZ\gamma$ (lowest) and 44 fb for $W^+W^-\gamma$ and $Z\gamma\gamma$ (highest). For comparison, the cross sections for $Z\gamma$ and $W^\pm\gamma$ production at 2 TeV are about 16 pb and 17 pb, respectively. Increasing the c. m. energy from 2 TeV to 4 TeV would enhance the production cross sections for the triple gauge-boson production by about a factor of 3. Naively, one would expect that the W^\pm -related cross sections should be larger than those of Z , like in the case of Drell-Yan

production of W^\pm and Z bosons, due to the weak charge-current coupling being stronger than the neutral-current coupling. It is therefore surprising to see that $W^\pm\gamma\gamma$ rate is even smaller than that of $Z\gamma\gamma$. In fact, this is due to some severe gauge cancellation in $W^\pm\gamma\gamma$ process, a phenomenon called radiation amplitude zero [30]. The fact that the $W^\pm Z\gamma$ cross section tends to be somewhat smaller than naively expected (compared to $W^+W^-\gamma$ and $ZZ\gamma$ cases) may be also traced to a radiation amplitude zero for $W^\pm\gamma$ process [30] and an approximate zero in $W^\pm Z$ process [31].

Figure 2 presents the total cross sections for processes with three massive bosons, Eq. (3). The cross sections are generally smaller by about an order of magnitude if we replace a photon [with our choice of the cuts in Eq. (5)] by a Z . The cross section rates at $\sqrt{s}=2$ TeV are about 0.5 fb for ZZZ process (lowest) and 6 fb for $W^+W^-W^\pm$ process (highest). Again for comparison, the cross sections for ZZ , $W^\pm Z$ and W^+W^- production at 2 TeV are about 1.1 pb, 2.5 pb and 9.2 pb, respectively. It is interesting to note that the Higgs boson in the SM contribute to the processes of Eq. (3) through $H^0 \rightarrow VV$. In the calculation for Fig. 2, we have taken the Higgs boson mass $m_H = 100$ GeV, for which there is no significant contribution from H^0 in VVV production. If the Higgs boson mass is just above the VV pair threshold, the enhancement to VVV production could be sizeable. The Higgs boson effects are demonstrated in Fig. 3, where the total cross sections are plotted versus m_H for $\sqrt{s} = 2$ TeV. We see that the increase right above the threshold is as much as a factor of 4 for $W^\pm ZZ$ and ZZZ channels through $H^0 \rightarrow ZZ$, and a factor of 6 for $W^+W^-W^\pm$ and W^+W^-Z through $H^0 \rightarrow W^+W^-$.

For the top-quark induced triple gauge-boson production (via $t\bar{t} \rightarrow W^+W^-$ with a 100% branching fraction), the $t\bar{t}W^\pm$ production goes only through $q\bar{q}'$ -annihilation diagrams, in which one of the light quarks radiates a W ; while $t\bar{t}Z$ and $t\bar{t}\gamma$ get contributions from both $q\bar{q}$ -annihilation and gg -fusion. At Tevatron energies, the valence quark contributions dominate and the gg -fusion only contributes a few percent. Cross sections for those channels are presented in Fig. 4. Formally, these processes are of order $\alpha_s^2\alpha$, compared to the electroweak triple gauge-boson cross section of order α^3 . However, due to phase space suppression from

the large top-quark mass, the production cross sections in Fig. 4 are comparable to that of $W^+W^-\gamma$ in Fig. 1 and those of $W^+W^-W^\pm$ and W^+W^-Z in Fig. 2, although the rates for the $t\bar{t}V$ processes increase more rapidly than those of VVV for higher energies. Since the heavy top quark often gives a hard b -jet as a decay product, one may be able to separate the electroweak W^+W^-V process from $t\bar{t}V \rightarrow W^+W^-V$ by examining the additional jet activities, if desired. We have not included the processes resulting from $H^0 \rightarrow t\bar{t}$, such as

$$p\bar{p} \rightarrow W^\pm H^0 \rightarrow W^\pm t\bar{t} \quad \text{and} \quad p\bar{p} \rightarrow ZH^0 \rightarrow Zt\bar{t}. \quad (6)$$

The decay branching fraction of $H^0 \rightarrow t\bar{t}$ is never larger than those of $H^0 \rightarrow VV$ which we included in the calculations for Eq. (3). Also, the cross sections of Eq. (6) would be large only if $m_H \geq 2m_t \simeq 400$ GeV, where the total rate at Tevatron energies must be very small due to the phase space suppression.

III. DISCUSSIONS

The predicted cross sections for triple gauge-boson production in the SM, as shown in Figs. 1–4, at Tevatron energies may result in a sizeable number of events if the luminosity is sufficiently high. With the Tevatron Main Injector, an annual integrated luminosity of order 1 fb^{-1} is envisioned. A further upgrade of the luminosity to reach $10 \text{ fb}^{-1}/\text{year}$ may become reality [9,10]. However, in the hadron collider environment, the large QCD backgrounds may render the observation impossible for those events if one or more of the gauge bosons decay hadronically. Individual channels with hadronic decays should be studied on a case by case basis. For simplicity, we only discuss the cleanest channels with pure leptonic decays of the gauge bosons. Taking the relevant branching fractions to be [28]

$$BR(W^+ \rightarrow l^+\nu) = 21\%, \quad BR(Z \rightarrow l^+l^-) = 6.7\%, \quad (7)$$

where $l = e$ and μ , we have

$$\begin{aligned} BR(W^+W^- \rightarrow l^+\nu l^-\bar{\nu}) &= 4.6\%, \quad BR(ZZ \rightarrow l^+l^-l^+l^-) = 0.45\%, \\ BR(W^+Z \rightarrow l^+\nu l^+l^-) &= BR(W^-Z \rightarrow l^-\nu l^+l^-) = 1.4\%, \end{aligned} \quad (8)$$

and

$$\begin{aligned} BR(W^+W^-W^+ \rightarrow l^+\nu l^-\bar{\nu}l^+\nu) &= 0.98\%, \quad BR(W^+W^-Z \rightarrow l^+\nu l^-\bar{\nu}l^+l^-) = 0.31\%, \\ BR(W^+ZZ \rightarrow l^+\nu l^+l^-l^+l^-) &= 9.7 \times 10^{-4}, \quad BR(ZZZ \rightarrow l^+l^-l^+l^-l^+l^-) = 3.0 \times 10^{-4}. \end{aligned} \quad (9)$$

For the direct photons, we assume that they can be identified effectively as isolated showers in the electromagnetic calorimeter.

With these branching fractions, one can easily estimate the cross section rate for a given leptonic mode based on the total cross sections presented in Figs. 1–4. We summarize the cross section rates in units of fb for all channels in two Tables for $\sqrt{s}=2$ TeV. Table I presents the processes with direct photons and Table II only those with massive V 's. Entries in the first row list the total cross sections read off directly from the figures; those in the second row give the cross section rates with the relevant leptonic branching fractions. The letter X there denotes the missing neutrinos. We see that with an integrated luminosity of 10 fb^{-1} , one would expect a few dozen of pure leptonic events for $W^\pm\gamma\gamma$, $Z\gamma\gamma$ and $W^+W^-\gamma$, making the studies for anomalous quartic couplings at the upgraded Tevatron possible. The other channels are of very small event rate, even though a Higgs boson with $m_H \geq 2M_V$ could enhance the VVV production rates by a factor 4 – 6. The trilepton channels of $W^+W^-W^\pm$ and $t\bar{t}W^\pm \rightarrow l^+l^-l^\pm X$ are of special interest since they are the backgrounds to the benchmark SUSY processes for gaugino signals. The total rate of about 0.1 fb shown in Table II would not seem to pose a severe problem for the trilepton SUSY signals [17].

We have thus far simply imposed a minimal η^γ -cutoff (< 2.5) on the final state photons. It is informative to examine the photon pseudorapidity distribution. Figure 5(a) shows the $|\eta^\gamma|$ distribution for $W^+W^-\gamma$, $Z\gamma\gamma$ and $W^\pm\gamma\gamma$ processes, and Fig. 5(b) presents the $|\eta_{\text{max}}^\gamma|$ (the larger one of the two $|\eta^\gamma|$) distribution for $W^\pm\gamma\gamma$ and $Z\gamma\gamma$ processes, with a cut $p_T^\gamma > 10$ GeV. We see that if we tighten up the η^γ -cut to be 1, the total rates would be reduced to 22 fb (by 49%) , 13 fb (73%) and 5.4 fb (75%) for $W^+W^-\gamma$, $Z\gamma\gamma$ and $W^\pm\gamma\gamma$ processes, respectively (cf. Table I).

It is important to note that in Fig. 5(a) the cross section rate for $W^\pm\gamma\gamma$ in the central

scattering region (small pseudorapidity) is much lower than that for $Z\gamma\gamma$ and $W^+W^-\gamma$: a direct reflection of the radiation amplitude zeros in the $W^\pm\gamma\gamma$ processes. When the two photons are collinear, there would be an exact radiation amplitude zero [30] in the parton c. m. frame. For the dominant process $u_1\bar{d}_2 \rightarrow W^+\gamma\gamma$, it is located at

$$\cos\theta^\gamma = -\frac{Q_u + Q_d}{Q_u - Q_d}, \quad (10)$$

which is at $-1/3$; while the zero would be located at $+1/3$ for $d_1\bar{u}_2 \rightarrow W^-\gamma\gamma$. It is these zeros that make the $W^\pm\gamma\gamma$ cross section small in the central region. More importantly, the zeros are due to subtle gauge cancellation in the SM and thus sensitive to any anomalous couplings beyond the SM. We will examine this aspect in more detail in another publication.

Similar to Fig. 5, Figure 6 presents the transverse momentum distribution of the photon(s) for these three processes, with a cut $|\eta^\gamma| < 2.5$. As expected, the distributions fall off sharply.

Finally, we present the maximum rapidity distribution $|y_{\max}|$ for the massive gauge-bosons for two generic processes, $p\bar{p} \rightarrow W^+W^-W^\pm$ and $p\bar{p} \rightarrow W^+W^-Z$ in Fig. 7. We see that the gauge-bosons are produced in the central region, with rapidities typically of $|y| \leq 1.5$. The charged leptons from the V decays then should be mostly within a rapidity range less than 2.5. Also, the transverse momenta for the final state charged leptons will roughly peak around $M_V/2$ as the Jacobian peak for the two-body V -decay. Therefore, we would not expect to lose much rate after imposing realistic leptonic cuts.

IV. SUMMARY

The triple gauge-boson production processes must be quantified for future experiments at an upgraded Tevatron in order to explore physics beyond the SM, such as studying quartic gauge-boson self-interactions, and searching for new particles like gauginos, gluinos and squarks in SUSY theories with leptonic final states. We have calculated the cross sections for the triple gauge-boson production in the Standard Model (SM) at Fermilab Tevatron

energies. At $\sqrt{s} = 2$ TeV, the cross sections are typically of order 10 fb. We have also estimated the production rates for multi-lepton final states from the gauge-boson decays. The cross sections for $p\bar{p} \rightarrow W^\pm\gamma\gamma, Z\gamma\gamma$ and $W^+W^-\gamma$ processes to pure leptonic final states at $\sqrt{s} = 2$ TeV are of order a few fb, resulting in a few dozen clean leptonic events for an integrated luminosity of 10 fb^{-1} , with the minimal cuts of Eq. (5) on the final state photons. The pure leptonic modes from other gauge-boson channels give significantly smaller rate. Especially, the trilepton modes yield a cross section of order 0.1 fb at 2 TeV if there is no significant Higgs boson contribution. For $m_H \simeq 2M_V$, the VVV production rates could be enhanced by a factor of 4 – 6.

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FIGURES

FIG. 1. Total cross sections in units of fb versus the c. m. energy \sqrt{s} for $p\bar{p} \rightarrow V\gamma\gamma$ and $p\bar{p} \rightarrow VV\gamma$ production (where $V = W^\pm, Z$). Minimal cuts of Eq. (5) on the final state photons have been imposed.

FIG. 2. Total cross sections in units of fb versus the c. m. energy \sqrt{s} for $p\bar{p} \rightarrow VVV$ production. Here $m_H = 100$ GeV is used.

FIG. 3. Total cross sections in units of fb versus the Higgs boson mass m_H for $p\bar{p} \rightarrow VVV$ production at $\sqrt{s} = 2$ TeV.

FIG. 4. Total cross sections in units of fb versus the c. m. energy \sqrt{s} for $p\bar{p} \rightarrow t\bar{t}V$ production. Here $m_t = 175$ GeV is assumed.

FIG. 5. Photon pseudorapidity distribution at $\sqrt{s}=2$ TeV for (a). $d\sigma/d\eta^\gamma$ and (b). $d\sigma/d\eta_{\max}^\gamma$ with $p_T^\gamma > 10$ GeV.

FIG. 6. Photon transverse momentum distribution at $\sqrt{s}=2$ TeV for (a). $d\sigma/dp_T^\gamma$ and (b). $d\sigma/dp_{T\min}^\gamma$ with $|\eta^\gamma| < 2.5$.

FIG. 7. Maximum rapidity distribution $d\sigma/dy_{\max}$ for the massive gauge-bosons at $\sqrt{s}=2$ TeV. Here $m_H = 100$ GeV is used.

TABLES

TABLE I. Cross sections in units of fb for triple gauge-boson production in $p\bar{p}$ collisions for $\sqrt{s}=2$ TeV and $m_t = 175$ GeV. Minimal cuts of Eq. (5) on final state photons have been imposed. The entries in the bottom row include the pure leptonic branching fractions given in Eq. (7-8), and X denotes the missing neutrinos.

$W^\pm\gamma\gamma$	$Z\gamma\gamma$	$W^+W^-\gamma$ ($t\bar{t}\gamma \rightarrow W^+W^-\gamma$)	$W^\pm Z\gamma$	$ZZ\gamma$
21	48	44 (36)	8.2	6.9
$l^\pm\gamma\gamma X$	$l^+l^-\gamma\gamma$	$l^+l^-\gamma X$ ($t\bar{t}\gamma \rightarrow l^+l^-\gamma X$)	$l^\pm l^+l^-\gamma X$	$l^+l^-l^+l^-\gamma$
4.6	3.2	2.0 (1.7)	0.12	3.1×10^{-2}

TABLE II. Cross sections in units of fb for triple gauge-boson production in $p\bar{p}$ collisions for $\sqrt{s}=2$ TeV, $m_t = 175$ GeV and $m_H = 100$ GeV. The entries in the bottom row include the pure leptonic branching fractions given in Eq. (9), and X denotes the missing neutrinos.

$W^+W^-W^\pm$ ($t\bar{t}W^\pm \rightarrow W^+W^-W^\pm$)	W^+W^-Z ($t\bar{t}Z \rightarrow W^+W^-Z$)	$W^\pm ZZ$	ZZZ
6.0 (5.4)	4.9 (3.0)	1.1	0.49
$l^+l^-l^\pm X$ ($t\bar{t}W^\pm \rightarrow l^+l^-l^\pm X$)	$l^+l^-l^+l^- X$ ($t\bar{t}Z \rightarrow l^+l^-l^+l^- X$)	$l^\pm l^+l^-l^+l^- X$	$l^+l^-l^+l^-l^+l^-$
5.9×10^{-2} (5.3×10^{-2})	1.5×10^{-2} (9.3×10^{-3})	1.1×10^{-3}	1.5×10^{-4}

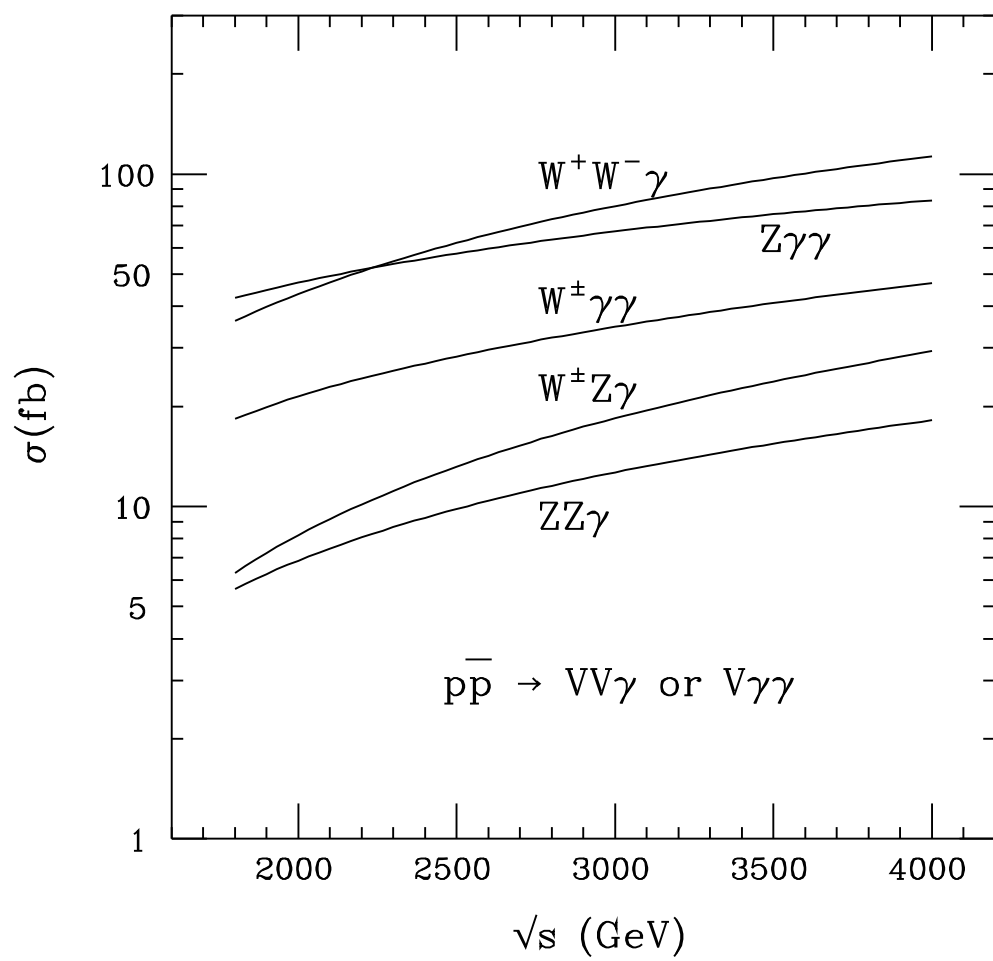


Figure 1

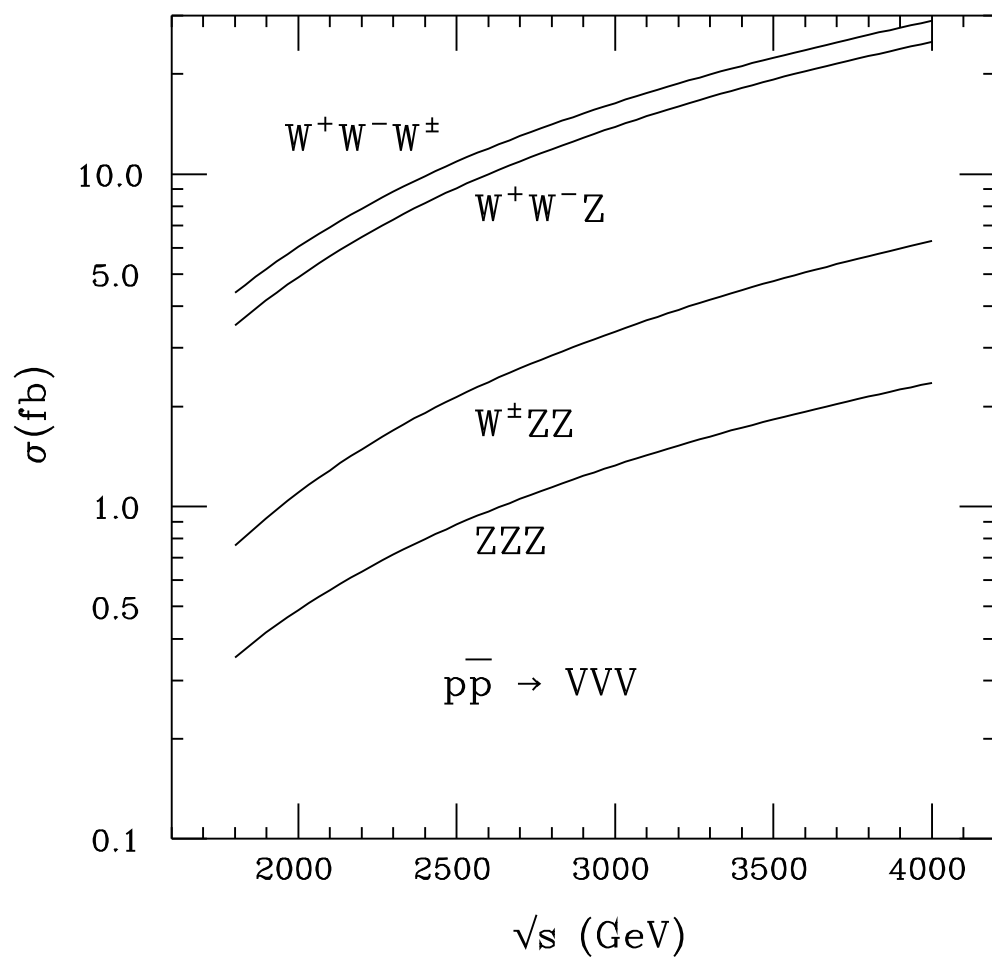


Figure 2

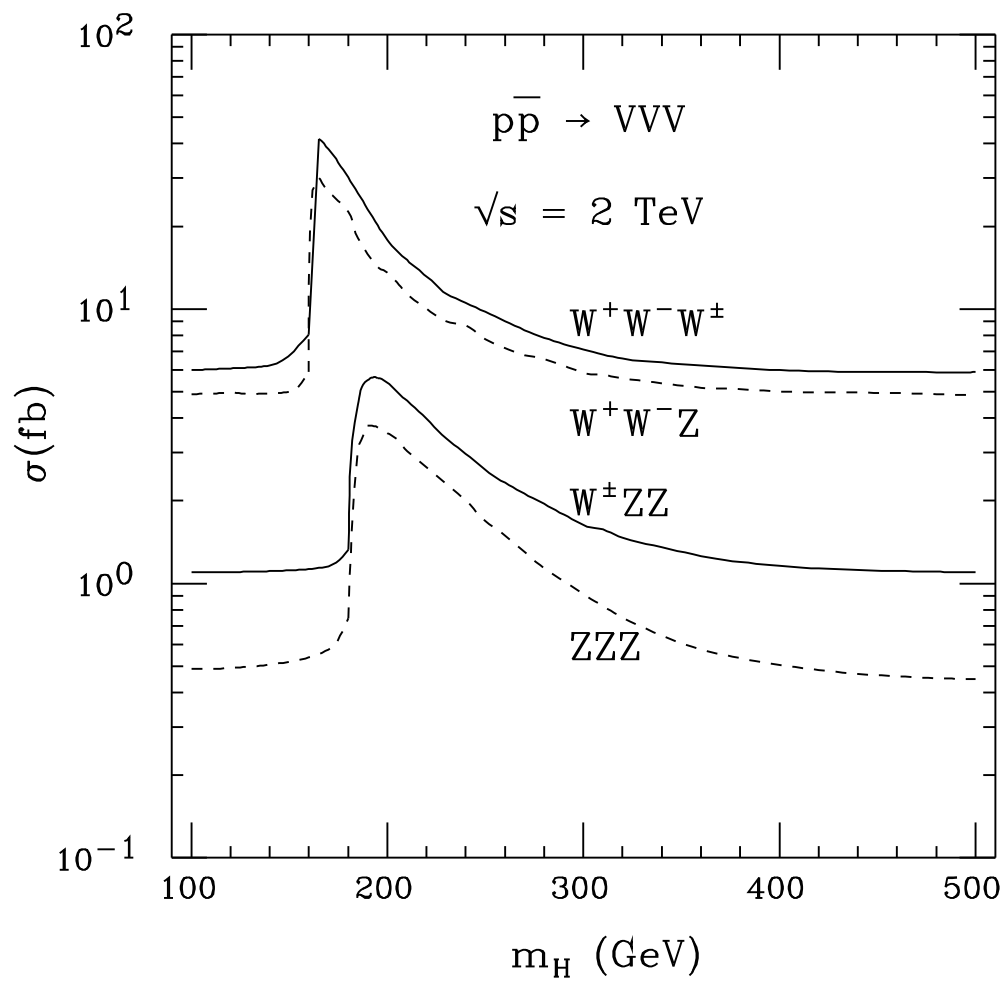


Figure 3

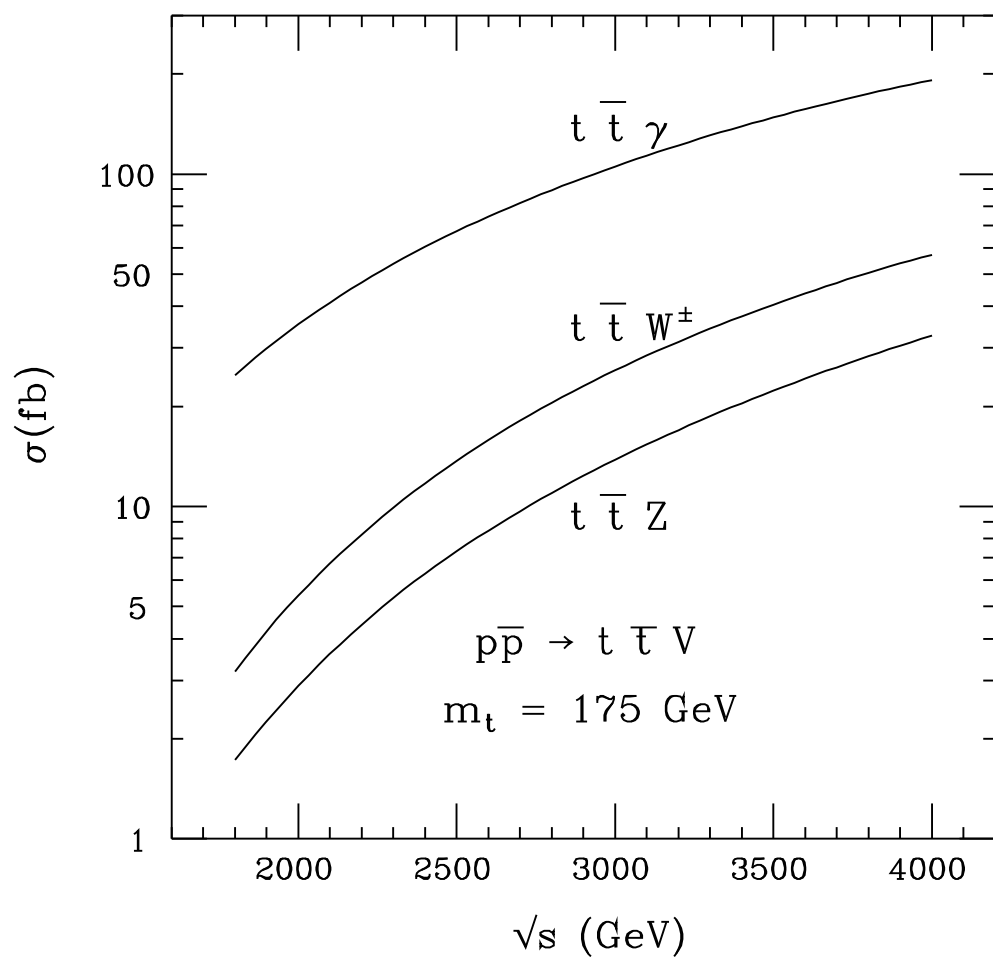


Figure 4

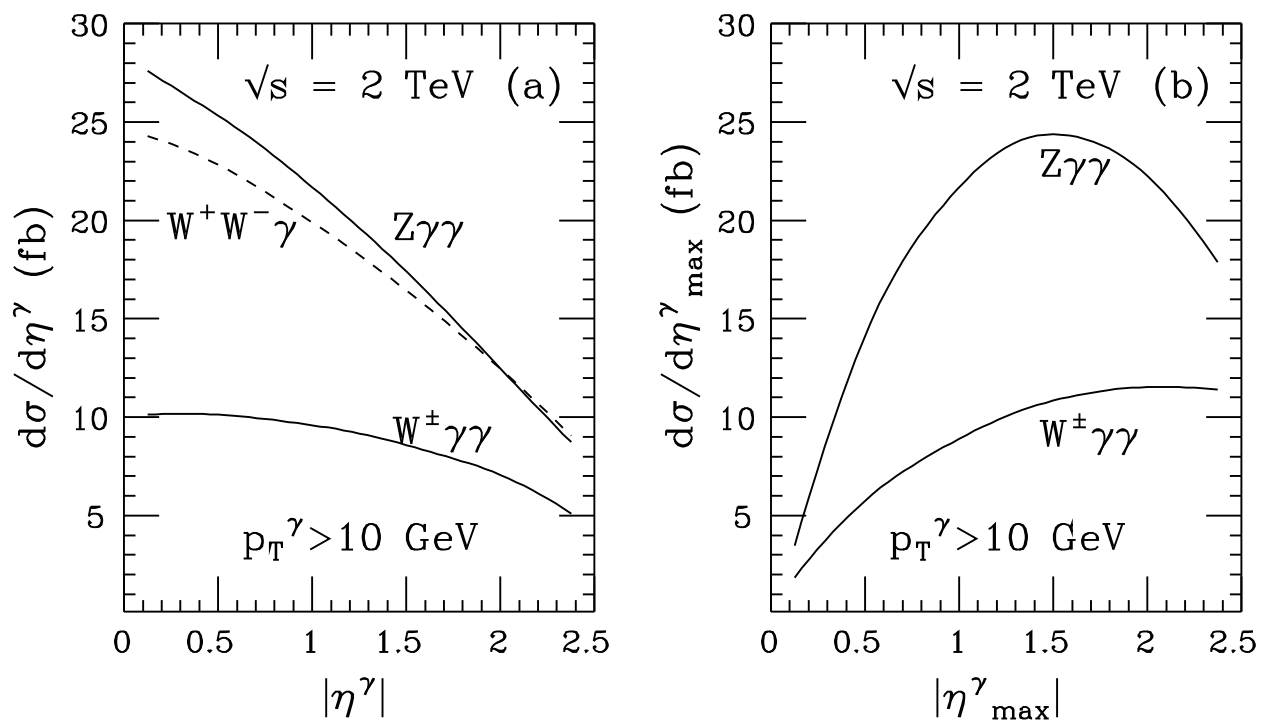


Figure 5

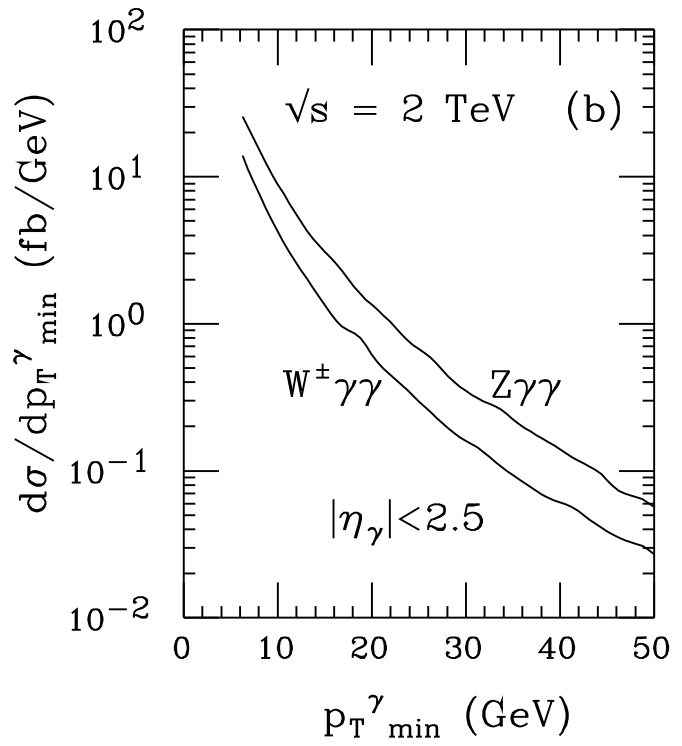
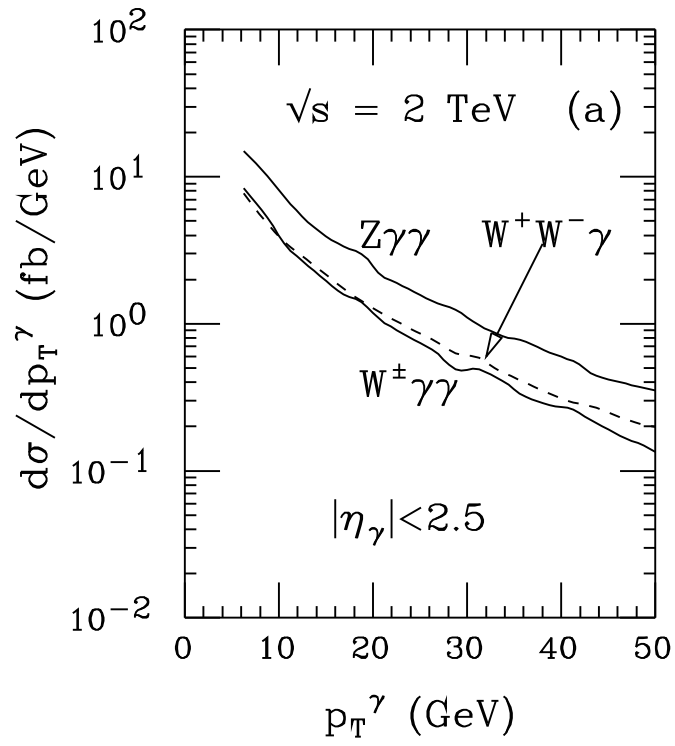


Figure 6

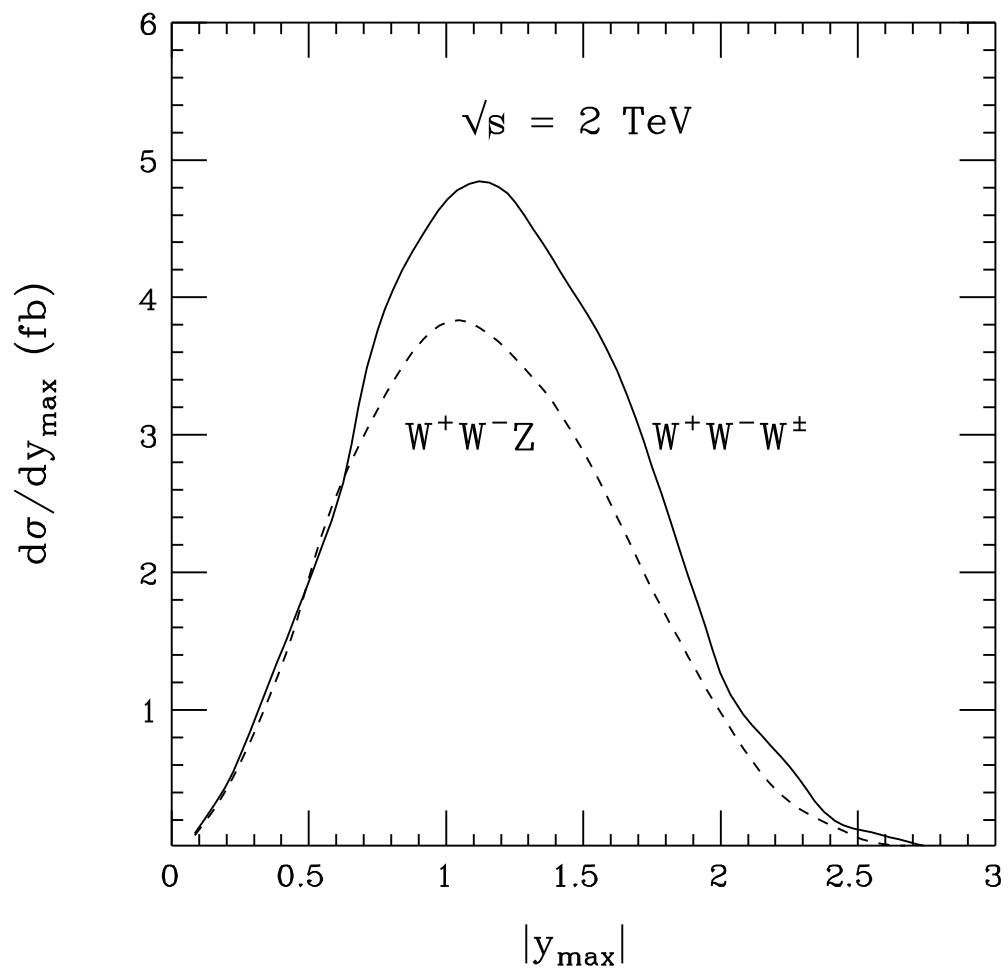


Figure 7